Broomwood Primary School

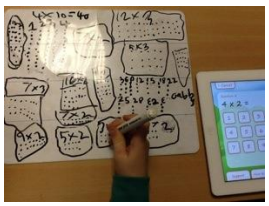
Guidance on Mental and Written Calculations

This policy shows a progression of calculations from Preschool through to Year Six. It is progressive and most steps are necessary for understanding to progress to the next method, however each cohort is different and the most important thing is that children have a good understanding of the four rules of number and how to approach them. It is important to teach children to question:

Which calculation method should I use?

- **Can I do it in my head using a mental strategy?**

- **Could I use jottings to help me?**

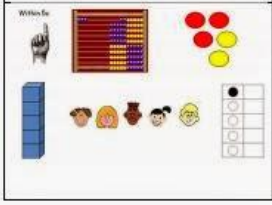
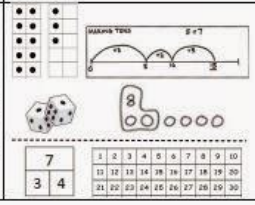
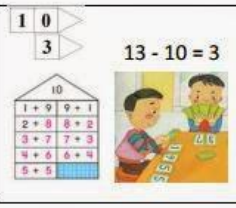


- **Should I use a written method to work it out?**

Children need practise in lessons to decide on how to best approach a question using the most efficient method. Can they explain why it is the most efficient method?

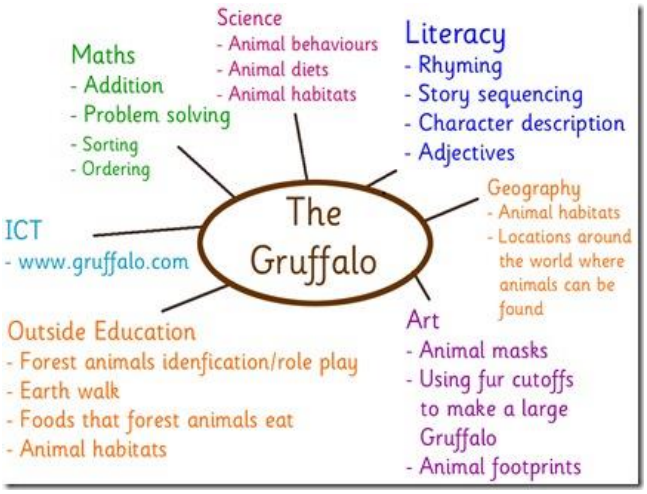
Each concept should be taught will all of these elements (age appropriate):

- Pre teach (appropriate to the stage in the year) or Baseline or observations
- Concrete (practical) learning
- Pictorial/visual/representational learning
- Abstract (written and symbol) learning/mark making (only when appropriate and they are ready)

Concrete	Representational	Abstract
Students manipulate hands-on, concrete materials	Students draw and observe diagrams, or watch the teacher touching and moving hands-on materials	Numbers and mathematical symbols
		

- Reasoning opportunities throughout and a whole class reasoning activity at the end
- Post learning or observations
- Revisit if not achieved or focus with a group that haven't achieved the objective.

Approach in EYFS – Pre School to Reception - is to immerse maths throughout continuous provision:
Choice of task: Think - resources and skilful questioning. The resources themselves may not be regarded as inherently mathematical, but will create a rich context for our children to develop problem-solving skills.
Choice of approach: Think – follow, support, question, stimulate thinking.
The structure we follow as adults is:
Describe – Prompt the children to talk and use mathematical language
Record – To communicate, explore or record what they have done
Reason – Prompt the children to explain why and connect ideas together
Open out – Exploring more deeply.



<u>Calculations should be taught in this order:</u>

Addition
Multiplication
Subtraction
Division

Subtraction and multiplication can be swapped where necessary.

This is what the children in Year Six should be able to do, to know where we are working towards:

https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/238967/Mathematics_Appendix_1.pdf

Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division

This appendix sets out some examples of formal written methods for all four operations to illustrate the range of methods that could be taught. It is not intended to be an exhaustive list, nor is it intended to show progression in formal written methods. For example, the exact position of intermediate calculations (superscript and subscript digits) will vary depending on the method and format used.

For multiplication, some pupils may include an addition symbol when adding partial products. For division, some pupils may include a subtraction symbol when subtracting multiples of the divisor.

Addition and subtraction

789 + 642 becomes

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 7\ 8\ 9 \\ +\ 6\ 4\ 2 \\ \hline 1\ 4\ 3\ 1 \end{array}$$

Answer: 1431

874 – 523 becomes

$$\begin{array}{r} \\ 8\ 7\ 4 \\ -\ 5\ 2\ 3 \\ \hline 3\ 5\ 1 \end{array}$$

Answer: 351

932 – 457 becomes

$$\begin{array}{r} {\scriptstyle 8\ 12\ 1\ \ } \\ 9\ \not{3}\ 2 \\ -\ 4\ 5\ 7 \\ \hline 4\ 7\ 5 \end{array}$$

Answer: 475

932 – 457 becomes

$$\begin{array}{r} {\scriptstyle \ \ 1\ \ 1\ \ } \\ 9\ 3\ 2 \\ -\ \not{4}\ \not{5}\ 7 \\ \hline 4\ 7\ 5 \end{array}$$

$$\begin{array}{r} {\scriptstyle \ \ 5\ \ 6\ \ } \end{array}$$

Answer: 475

Short multiplication

24 × 6 becomes

$$\begin{array}{r} \\ 2\ 4 \\ \times\ \ \ 6 \\ \hline 1\ 4\ 4 \\ {\scriptstyle \ \ 2\ \ \ } \end{array}$$

Answer: 144

342 × 7 becomes

$$\begin{array}{r} \\ 3\ 4\ 2 \\ \times\ \ \ \ 7 \\ \hline 2\ 3\ 9\ 4 \\ {\scriptstyle \ \ 2\ 1\ \ } \end{array}$$

Answer: 2394

2741 × 6 becomes

$$\begin{array}{r} \\ 2\ 7\ 4\ 1 \\ \times\ \ \ \ \ \ 6 \\ \hline 1\ 6\ 4\ 4\ 6 \\ {\scriptstyle \ \ 4\ 2\ \ \ \ } \end{array}$$

Answer: 16 446

Long multiplication

24 × 16 becomes

$$\begin{array}{r} {\scriptstyle 2} \\ 24 \\ \times\ 16 \\ \hline 240 \\ 144 \\ \hline 384 \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 124 \\ \times\ 26 \\ \hline 2480 \\ 744 \\ \hline 3224 \\ {\scriptstyle 1\ 1} \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 124 \\ \times\ 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ {\scriptstyle 1\ 1} \end{array}$$

Answer: 3224

Short division

98 ÷ 7 becomes

$$\begin{array}{r} 14 \\ {\scriptstyle 2} \\ 7\overline{)98} \end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r} 86\ r2 \\ {\scriptstyle 3} \\ 5\overline{)432} \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r} 45\ r1 \\ {\scriptstyle 5} \\ 11\overline{)496} \end{array}$$

Answer: $45\frac{1}{11}$

Long division

432 ÷ 15 becomes

$$\begin{array}{r} 28\ r12 \\ 15\overline{)432} \\ 300 \\ \hline 132 \\ 120 \\ \hline 12 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r} 28 \\ 15\overline{)432} \\ 300 \quad 15\times20 \\ \hline 132 \\ 120 \quad 15\times8 \\ \hline 12 \end{array}$$

$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r} 28{\cdot}8 \\ 15\overline{)432{\cdot}0} \\ 300 \\ \hline 132 \\ 120 \\ \hline 120 \\ 120 \\ \hline 0 \end{array}$$

Answer: 28·8

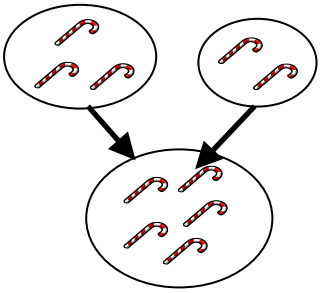
Mental Calculations: Addition

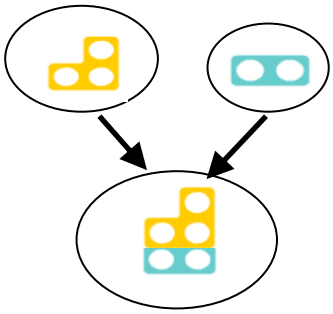
FS

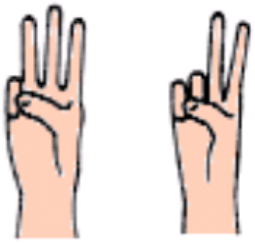
Stage 1: FIRST STEPS IN ADDITION

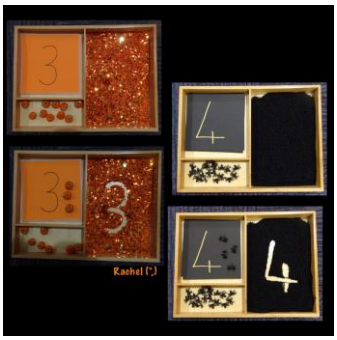
- Uses developing mathematical ideas and methods to solve practical problems involving **counting and comparing** in a real or role play context, using models and images to support learning.
- Is developing ways of **recording calculations using pictures, fingers, number tracks and bead strings** etc.

E.g. *3 + 2 = 5*









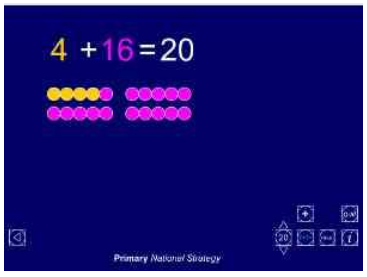
- Can say how many there are when **two groups of objects are combined** to make a total; extend to three groups using the imagery of combining sets to develop understanding.
- **Begins to use vocabulary** involved in addition (more, and, add, make, sum, total, altogether, score… one more, two more…, how many more to make….? How many more is… than…?)
- Can answer questions such as:

  What is this number? Find it. Which is more: 4 or 7?

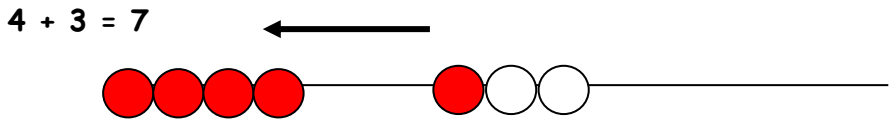
  What number: comes after 10, is one more than 5, 12? etc.
- **Begins to relate addition to counting on.**
- May begin to count on from the larger number.

Stage 2: DEVELOPING UNDERSTANDING

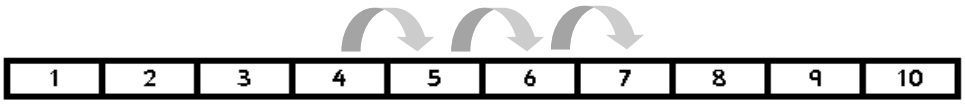
- Uses **related vocabulary and symbols to describe and record addition number sentences**.
- **Vocabulary** involved in addition (more, and, add, make, sum, total, altogether, score… one more, two more…, how many more to make….? How many more is… than…?, equal to, fewer, more than, less than)
- Recognises that **addition can be done in any order** to carry out mental calculations more efficiently.
- **Puts the larger number first** and **counts on in ones** (stepping), including beyond 10 and up to 100, using the bead string or labelled number line. Number bonds to 10 and 20.



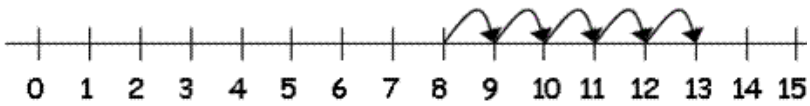
E.g. using a bead string: 4 + 3 = 7 and 32 + 1 = 33



E.g. on a number track: 4 + 3 = 7 and 17 + 3 = 20



E.g. on a number line: 8 + 5 = 13 and 18 + 2 = 20



- Is beginning to **interpret situations as addition calculations** and explain reasoning, for example, can answer questions such as:

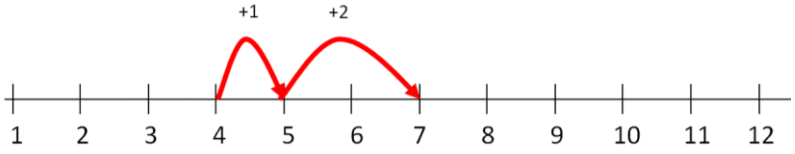
  Megan has 9 pens and Tom has 7 pens. How many pens do they have altogether?

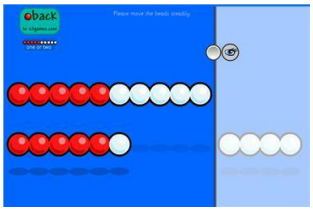
- Understands that more than two numbers can be added together.
- **Adding zero to a number.**

Stage 3: PARTITIONING

- Is developing their recall of number facts, including knowledge of '5 and a bit' partitioning.
- Can add a pair of numbers by **jumping** (not always stepping) along a number line, using knowledge of partitioning numbers in different ways.

E.g. 4 + 3 = 7 using knowledge of bridging through 5



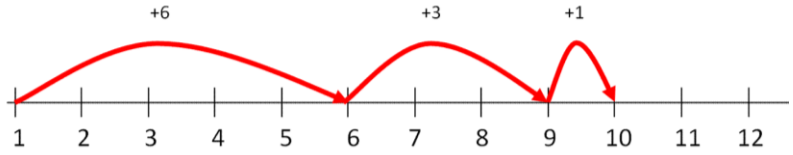


- Is able to use both **hands** to show addition calculations involving numbers to 10, including using the '5 and a bit' structure (as in 7 = 5 + 2). ***NB. these calculations should be reinforced using the bead string, tens frame and other imagery.***

E.g. 'Show me' games where 6 can be made on hands as 3 + 3, 4 + 2 and 5 + 1 (with the last example being the '5 and a bit' structure).

- **Understands that more than two numbers can be added together**, e.g. explore three jumps to 10 (or any other suitable number).

E.g. 6 + 3 + 1 = 10



Next steps: Bridging through 10.

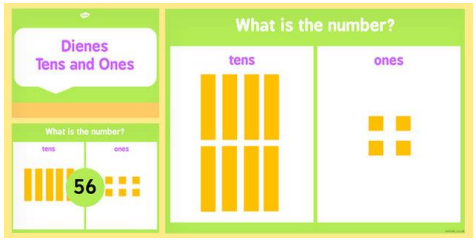
E.g. 8 + 5 = 13 using knowledge of number bonds to 10.

Next steps: Children become more efficient by using known number facts to count on.

E.g. 8 + 5 = 13 using knowledge of number bonds to 10 AND the number fact 2 + 3 = 5.

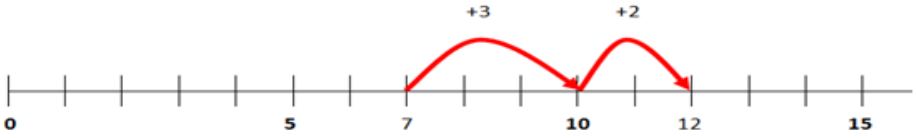
NOTE: When children are reliably jumping, begin to remove some of the support provided by the fully marked and labelled number line (e.g. leaving only the multiples of 5 labelled) working towards using a blank number line.

Stage 4: EXTENDING PARTITIONING AND ROUNDING AND ADJUSTING



- Can add a pair of numbers mentally by jumping and **bridging** through 10 and with numbers to at least 100.

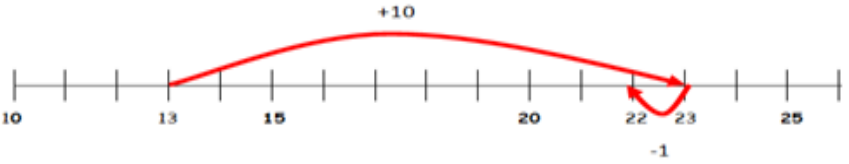
  E.g. for 7 + 5 = 12 by jumping along a number line



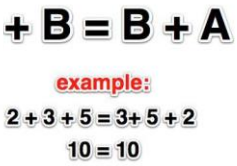
  Next steps: Bridging through 20, and on to other multiples of 10 *E.g. 48 + 4 = 52*

- Uses the method of **rounding and adjusting**: add a multiple of 10 and adjust (sometimes known as compensation).

  E.g. for 13 + 9 = 22, calculate 13 + 10 - 1

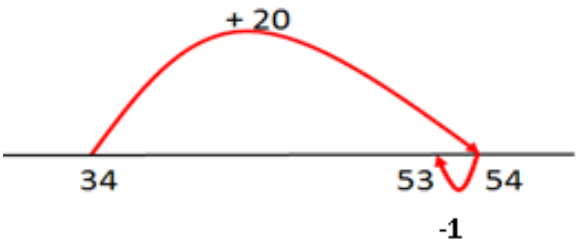


- To investigate that addition is commutative

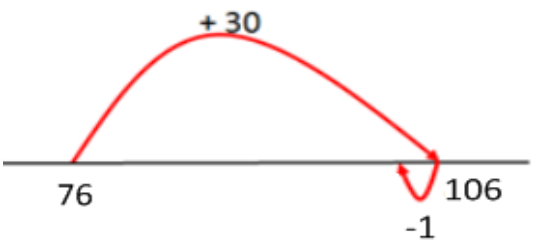


Progression in Rounding and Adjusting method (compensation)

E.g. 34 + 19 = 53

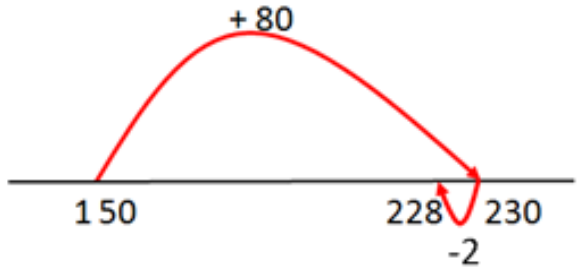


E.g. 76 + 29 = 105



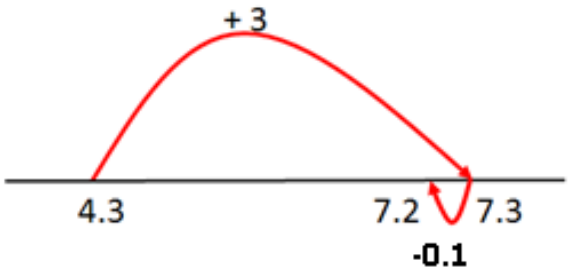
Year 2/3:
Children jump in multiples of ten to make the calculation more efficient up to 100/1000.
They begin to use the method for HTO + TO calculations.

E.g. 149 + 79 = 228

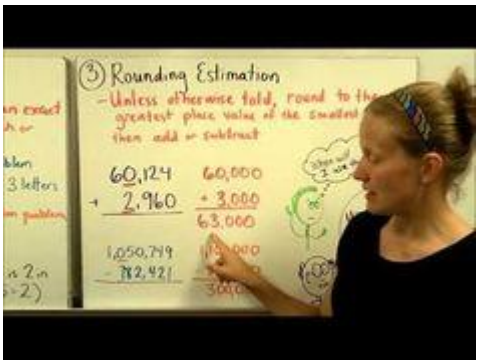


Year 4:
Children confidently use the method for HTO + HTO and can bridge through multiples of 100 beyond 1000.
To add tenths to O.t

E.g. 4.3 + 2.9 = 7.2



Year 5:
Children complete calculations involving more than one adjustment.
To add increasingly large numbers up to 1 000 000
To add decimals



Year 6:
Children complete addition with larger numbers and apply in a problem context.

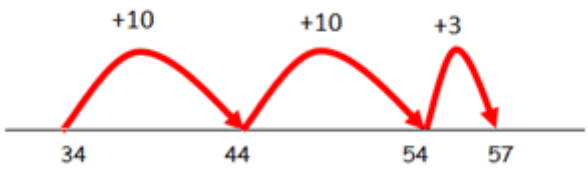
Stage 5: PARTITIONING INTO COMPONENT PARTS

Y2

- Uses a 100 square or empty number line to add two digit and ones, three 1 digit numbers and 2 two digit numbers.

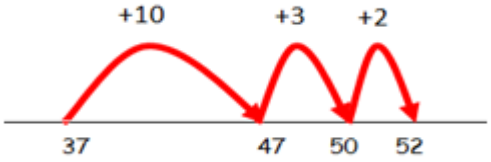
Method: Children partition the second number **only**. They add the tens to the first number. Then they add the ones.

E.g. 34 + 23 = 57 by partitioning into component parts on a number line



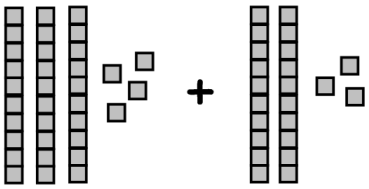
NB Bridging through 10s can help children become more efficient with this method:

E.g. 37 + 15 = 52 using partitioning and knowledge of bridging through tens



- Uses Dienes apparatus to add two 2-digit numbers by partitioning both numbers (no bridging).

E.g. 34 + 23 = 57 using Dienes apparatus



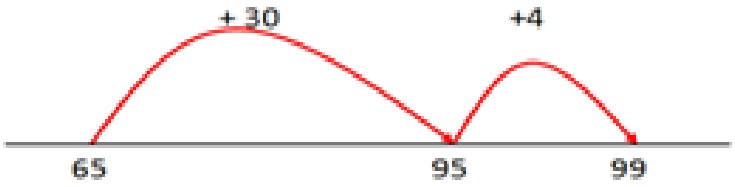
Next steps: Bridging tens by exchanging 10 ones for a ten. Then bridging hundreds.

- Introduce zero (0) as a place holder.

Progression in Partitioning method

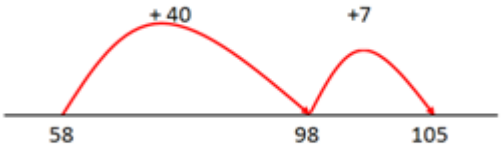
When there is nothing special about the numbers, children will use partitioning. On a number line, **they will use different sized jumps on the number line, depending on their level of confidence and efficiency**.

E.g. 34 + 65 = 99



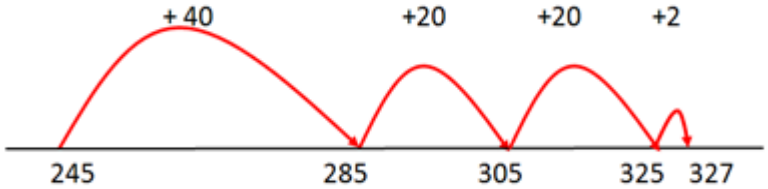
Year 2/3:
Children jump in multiples of ten to make the calculation more efficient.
They begin to use the method for HTO + TO calculations.

E.g. 47 + 58 = 105

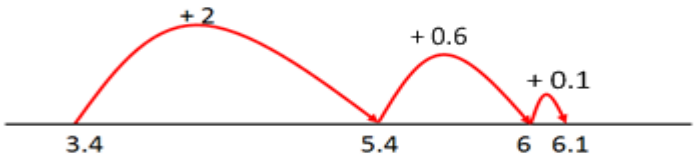


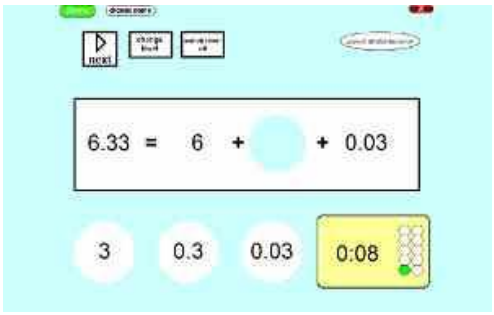
Year 4:
Children confidently use the method for HTO +TO and can bridge through multiples of 100 beyond 100.

E.g. 245 + 82 = 327



E.g. 2.7 + 3.4 = 6.1





Year 5:
Children confidently partition in a variety of ways using their knowledge of number facts and place value.

Year 6:
Children complete calculations involving decimals and use a variety of ways to be the most efficient.

FURTHER STEPS IN MENTAL ADDITION CALCULATIONS:

Children need to develop an understanding of the *most efficient mental method* to use by asking themselves the question: **Which mental method should I use?**
They look at the numbers in the calculation in order to make a decision about whether to use:

- **partitioning**
- **reordering** (using known facts or putting the larger number first)
- **using near doubles**
- **compensation** (rounding and adjusting)

Written Calculations: Addition

When using a written calculation method, children should be encouraged to:

- **approximate** their answers before calculating.
- **check** their answers after calculation using an appropriate strategy.

Children should continue to develop their use of empty number lines for recording their calculations and for calculations involving addition of 2 or more 3-digit sums of money or measures.

STAGE 6: INFORMAL EXPANDED METHOD

Y3

Introduce for: 3 digit + 2/3 digit addition.
NB Children should be using a mental method for 2 digit + 2 digit addition.

Method: Write the numbers in columns. Add the units first, then the tens etc.

```
  168
+  57
  ---
   15
  110
  100
  ---
  225
  ---
```

NB numbers are still added up mentally

NB. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and knowledge of partitioning.

STAGE 7: COMPACT METHOD

Y4

Method: The method doesn't change from the informal expanded method, but the recording is reduced and is extended up to 4 digits.

```
  168
+  57
  ---
  225
  ---
   11
```

'Carry' digits are recorded **below the line** explained as 'carry' ten or 'carry' hundred.

Two carries: 'units to tens' and 'tens to hundreds'

Next steps:

- Extend to numbers with any number of digits.
- When appropriate, use the compact method to add two or more decimal fractions with more than 4 digits (up to 1,000,000 in Y5 and 10,000,000 in Y6) and 1, 2 and 3 decimal places, including money and/or measures. Also missing numbers.

```
  2.65 kg
+ 1.47 kg
  ------
  4.12 kg
  ------
   11
```

Decimal points should be lined up: carry 'tenths to units' and 'units to tens'.

Y5/6

Mental Calculations: Multiplication

FS

Stage 1: FIRST STEPS IN MULTIPLICATION

Children will first approach multiplication in play and problem solving such as:

- making equal groups of objects.
- working on practical problem solving activities involving equal sets or groups.
- counting in 2s and 10s. Some children will then extend this to being able to count in 5s.

- They will learn how to respond, in practical situations, to questions such as:
  How many socks in two pairs?
  How many 10p coins are here? How much money is that?

Y1

Stage 2: DEVELOPING UNDERSTANDING

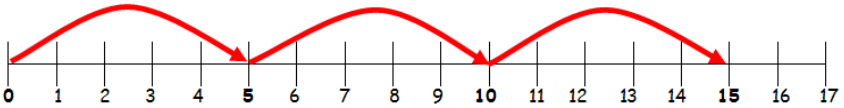
- Develops an understanding of multiplication as **repeated addition**. Children are able to use a range of models and images, drawings, and at times practical equipment, to represent repeated addition.

  E.g. Repeated addition using hands: 5 + 5 + 5 + 5 + 5 + 5 = 6 x 5 = 30

  E.g. Repeated addition using money: 10 + 10 + 10 + 10 + 10 = 5 x 10 = 50

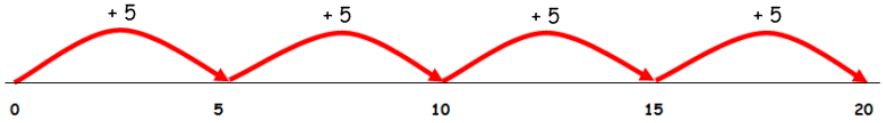
  E.g. Repeated addition using a bead string: 5 + 5 + 5 = 3 x 5 = 15

  E.g. Repeated addition using a number line: 5 + 5 + 5 = 3 x 5 = 15



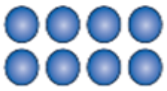
Y2

- As children **develop their recall of multiplication facts**, they then record **jumps** along an empty number line.
  E.g. Repeated addition using a blank number line: 5 + 5 + 5 + 5 = 4 x 5 = 20



- Develops an understanding of **arrays** (rows x columns).

Y2
cont…



2 **lots of** 4 = 2 x 4 = 8

4 **lots of** 2 = 8

- Begins to use **related vocabulary** (repeated addition, lots of, groups of, times, product of, multiplied by) and **symbols** to describe and record multiplication number sentences.

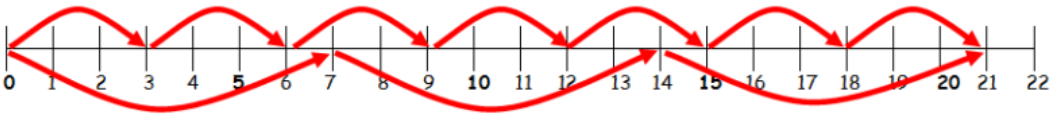
- Begins to **interpret situations as multiplication calculations**, and explain reasoning, for example:
  How many wheels are there on three cars?
  I have six 10p coins. How much money do I have altogether?

Stage 3: COMMUTATIVITY AND PARTITIONING

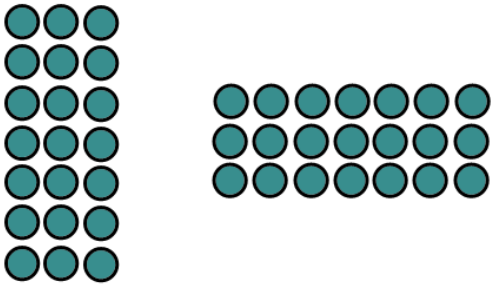
- Children are **introduced to the commutative law** through number lines and arrays.

E.g. they use images to show that 7 x 3 = 3 x 7 = 21.

On a number line:



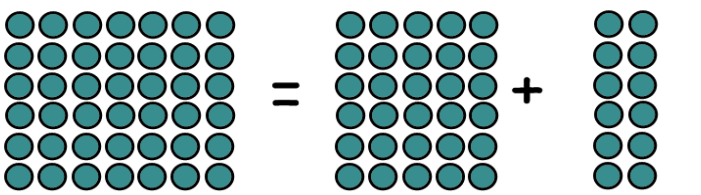
Using arrays:



- **Uses knowledge of commutativity and known facts** to calculate unknown multiplication facts, **e.g. uses 6 lots of 5 or 6 x 5 to calculate 5 lots of 6 or 5 x 6 = 30.**

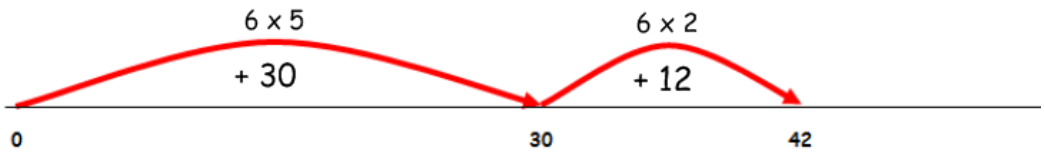
Y3

- Uses **known facts and partitioning** to work out unknown facts.
  E.g. calculate 6 x 7 using knowledge of 5 and 2 times table
  Using arrays:



6 x 7 = (6 x 5) + (6 x 2)
42 = 30 + 12

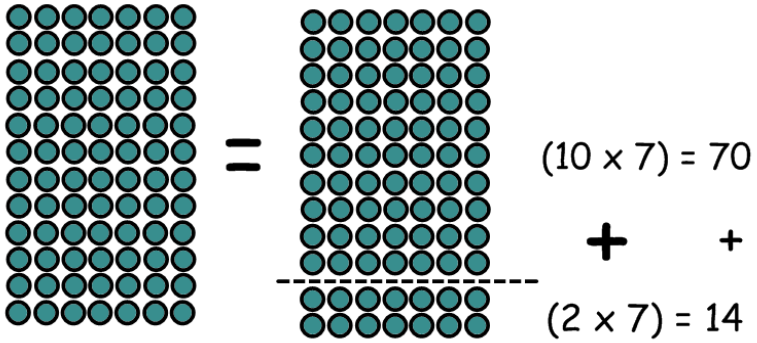
On a number line:



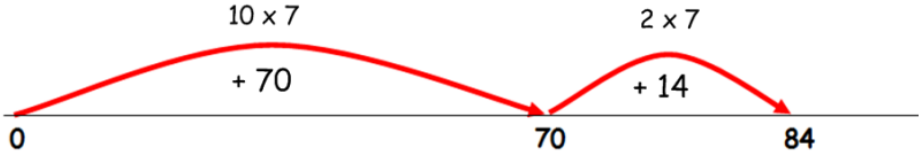
Next steps: Children progress to **partitioning into component parts**.

NB. a good understanding of this method will support children's development of the written method of grid multiplication.

E.g. calculate 12 x 7 by partitioning into component parts,
Using an array:



Using a number line:



Next steps: Once children are confident calculating on an empty number line, they can begin to record their mental calculations as jottings without the number line.
E.g. calculate 12 x 7 by partitioning into component parts, shown as informal jottings:

12 x 7 = (10 x 7) + (2 x 7)
= 70 + 14
= 84

- **Associates scaling up with multiplication.**
  E.g. find the length of a fence that is 4 times longer than:

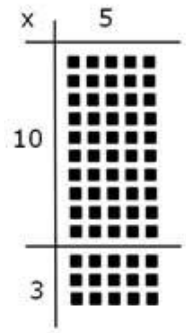
Length = 6 m

- **Count in multiples from 0 - 4, 8, 50 and 100 from a given number (Y3)**
- **Count in multiples from 0 - 6, 7, 9, 25 and 1000 (Y4)**

Written Calculations: Multiplication

Y3

STAGE 4: GRID MULTIPLICATION METHOD



3 8 x 7 = 2 6 6

x		7
30	210	
8		56
	266	

Short multiplication:
TO x O
NB. Children working at expected should be able to do this mentally as well as written

NB. Placing the number with the most digits on the left allows children to add the partial products easier, and also links to expectations for addition at this stage.

STAGE 5: INFORMAL EXPANDED METHOD

This method is a stepping stone between the grid method and the compact written method.

3 8 x 7 = 2 6 6

38
x 7
56
+ 210
266

Short multiplication:
TO x O

Y4

STAGE 6: COMPACT METHOD

This method should be extended to **HTO x O** and the compact method taught.

4 3 4 6 x 8 = 3 4 7 6 8

4346
x 8
34768
2 3 4

Short multiplication:
ThHTO x O

Y5

LONG MULTIPLICATION

Introduce for: TO x TO, HTO x TO and ThHTO x TO calculations.

3 5 2 x 2 7 = 9 5 0 4

			3	5	2					
		x		2	7					
		2	4	6	4					
		7	0	4	0					
		9	5	0	4					
			1							

Long multiplication: HTO x TO

Children need to be taught multiplying by both O first and T first to show multiplication is commutative and they can be confident both ways. If they are very secure with one way, they still need to be aware that they could do the other way also.

NB. The carry digits are carried below the line but crossed through once counted into the calculation to prevent confusion in the final column addition.

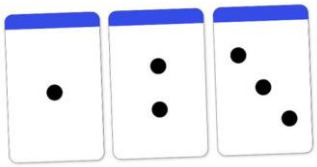
Next steps: Calculations involving decimal fractions.

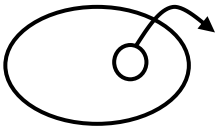
Mental Calculations: Subtraction

Stage 1: FIRST STEPS IN SUBTRACTION

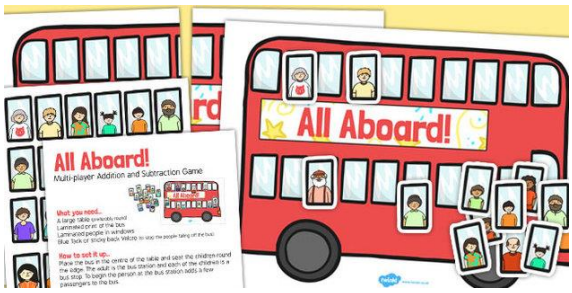
FS

- Uses developing mathematical ideas and methods to solve practical problems involving **counting and comparing** in a real or role play context using models and images to support learning.





- Carries out '**taking away**' following the reduction model using bead strings, number tracks and other mathematical imagery within number.
- **Begins to use vocabulary associated with subtraction** (take away, leave, how many are left? How many have gone? One less, two less, how many less is ...than...? What is the difference between?).
- Solves problems in everyday life in the classroom, or in role play. Make decisions about what to do. Explain orally and where appropriate, records the solution in their own way.



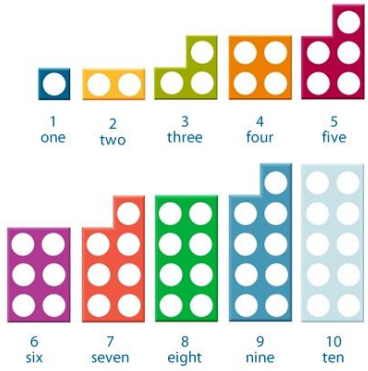
Y1

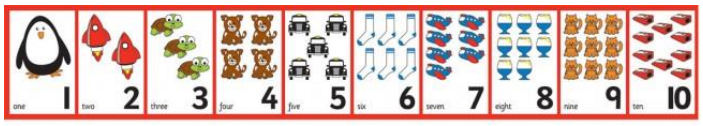
- Can answer questions such as:

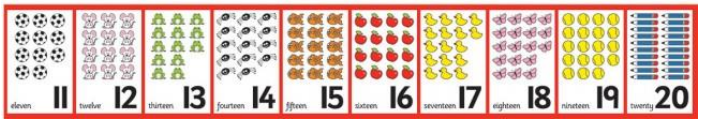
  What is this number? Find it. *Which is less: 4 or 7?*

  What number: comes before 10, is one less than 5, 12 etc.

- Says how many are left when some are taken away by counting how many are left, or by **counting back** from the number.
- Can find out how many have been removed or how many more will make a given number by **counting up** to the larger number.



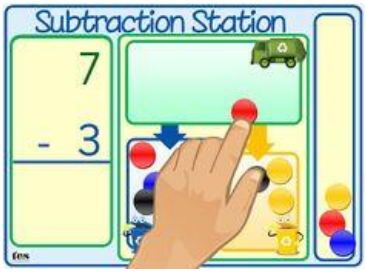




Stage

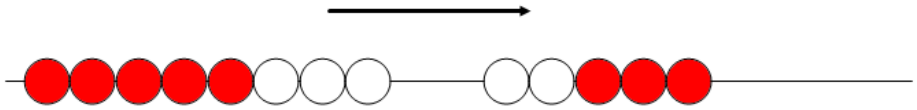
2: DEVELOPING UNDERSTANDING

- Uses related vocabulary (difference between, take away, minus, less than, fewer) and symbols to describe **and record subtraction number sentences**. Number bonds to 10 and 20.

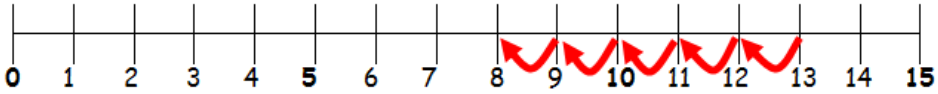


- Can **subtract one digit numbers from one and two digit numbers by stepping** along the bead string or labelled number line.

E.g. 13 — 5 = 8 using a bead string and 20 – 5 = 15



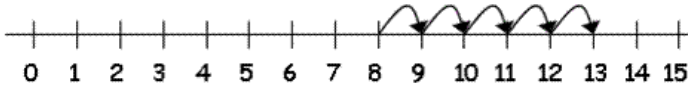
E.g. 13 — 5 = 8 by counting back in steps along a number line



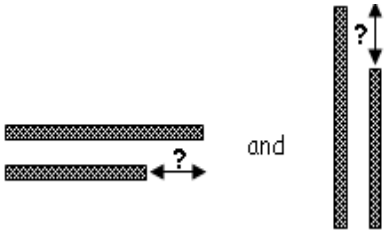
NB. <u>steps and jumps counting back</u> will be shown <u>under the line</u> to clearly differentiate between counting on and counting back.

- Understands and is beginning to use the method of **counting on** as one way to find the **<u>difference</u>** between numbers up to 100, especially if the difference is a small amount.

E.g. counting on in steps along a number line: 13 - 8 = 5



E.g. counting on by comparing amounts



- **Subtract zero from a number**
- Investigate that subtraction is the reverse of addition.
- Begins to **interpret situations as subtraction calculations** and explain reasoning. For example, can answer questions such as:

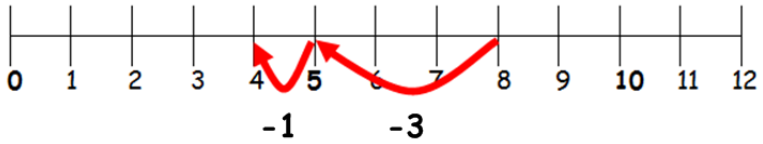
  There are 18 bean bags. Kim takes 5. How many bean bags are left?

  There are 9 people on a bus and 5 people in a car. How many more people are there on the bus than in the car?

Stage 3: PARTITIONING AND ROUNDING AND ADJUSTING

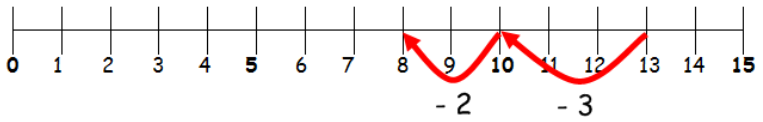
- Is developing their recall of number facts, including knowledge of '5 and a bit' partitioning.
- Subtracts a pair of numbers mentally by **jumping** on a number line.

  E.g. 8 – 4 = 4 by jumping on a number line:



  Next steps: Bridging through 10, then 20, other multiples of 10, then 100. .

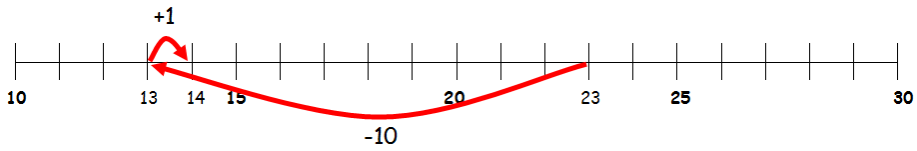
  E.g. 13 – 5 = 8 by bridging through 10 on a number line:



NOTE: When children are reliably jumping, begin to remove some of the support provided by the fully marked and labelled number line (e.g. only the multiples of 5 labelled) working towards using a blank number line.

- Uses **rounding and adjusting**: subtract a multiple of 10 and adjust by 1 (sometimes known as compensation)

  E.g. 23 – 9 by calculating 23 – 10 + 1 = 14



  Next steps: Work towards using an empty number line and subtracting multiples of 10 and adjusting.

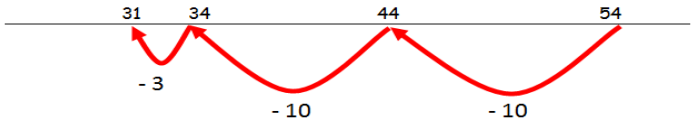
  - **Subtraction is not commutative**

Stage 4: PARTITIONING INTO COMPONENT PARTS

- Use a number line to **count back**.

  Method: Starting from the larger number, subtract the 'tens' first, as the most significant digit, then the units.

  E.g. 54 - 23 = 31 by partitioning and counting back on a number line



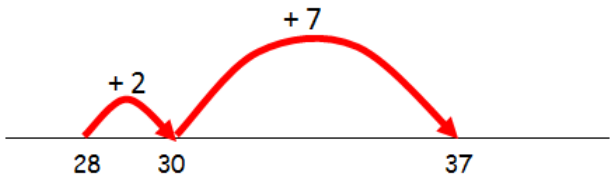
Next steps: Examples involving bridging through multiples of 10, and extending to larger numbers. Children can become more efficient in this method by jumping in multiples of 10, using recall of number facts to 10 and knowledge of place value. Can they show the reverse?

Y2/3

- Use a number line to **count on** to find the difference.

  Method: Starting at the smaller number, jump to the nearest 'ten' (or 'hundred' if appropriate), then count on to the larger number (the size of the jumps they use will depend on the children's knowledge of place value and basic number facts).

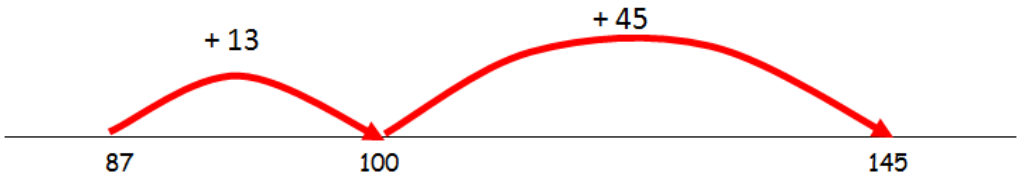
  E.g. 37 - 28 = 9 using partitioning with counting on, on a number line



Next steps: Find larger differences using knowledge of number bonds to 100 and understanding of place value. 3 digit subtract ones, 3 digit subtract Tens and 3 digits subtract 3 digits.

Y3

E.g. 145 - 87 = 58 using partitioning with counting on, on a number line



FURTHER STEPS IN MENTAL SUBTRACTION CALCULATIONS

Children need to develop an understanding of the *most efficient mental method* to use by asking themselves the question: **Which mental method should I use?**
They look at the numbers in the calculation in order to make a decision about whether to use:

- partitioning (and counting on *or* back)
- reordering (using known facts)
- compensation (rounding and adjusting)

NOTE: For the general method of partitioning it is important to discuss with the children whether to subtract by counting on or by counting back. Encourage them to ask themselves 'Which is the most efficient method for the numbers in the calculation?'
E.g. 45 - 12 would be counting back/take away whereas 87 - 72 would be counting on.

Written Calculations: Subtraction

Y2/3

STAGE 5: INFORMAL EXPANDED METHOD

In order to carry out this method, children must understand how to partition numbers in different ways.

E.g. partitioning 23 and 437 in different ways:

4	**3**	**7**	**=**	3	0	0	+	1	3	0	+ 7
			=	3	0	0	+	1	2	0	+ 1 7
			=	3	0	0	+	1	1	0	+ 2 7

Y3/4

Introduce for: HTO – TO calculations.

Method: Start with subtracting the ones, then the tens etc.

864 - 621 = 243

	8	0	0		6	0		4
-	6	0	0		2	0		1
	2	0	0	+	4	0	+	3

No adjustment

351 - 136 = 215

					40		11	
	3	0	0		~~5~~	0	~~1~~	
-	1	0	0		3	0		6
	2	0	0	+	1	0	+	5

One adjustment: discuss how the 51 has been partitioned into 40 and 11.

643 - 385 = 258

					130				
		500			~~3~~0		13		
	~~6~~	~~0~~	~~0~~		~~4~~	0	~~3~~		
-	3	0	0		8	0		5	
	2	0	0	+	5	0	+	8	

Two adjustments: discuss how the 643 has been partitioned into 500 + 130 + 13.

503 - 278 = 225

90

400 100 13

~~500~~ ~~0~~ ~~3~~

- 200 70 8

200 + 20 + 5

Dealing with zeros: OPTION 1

700 - 236 = 464

600 90 9 (+1)

- 200 30 6

400 + 60 + 3 (+1) = 464

Dealing with zeros: OPTION 2

Model to the children both methods to explain what is happening, but don't worry if they can't practise both. We just want them to think in different ways. Link to how we did partitioning in different ways for subtraction in Y2/3.

STAGE 6: COMPACT METHOD

Only children who can calculate independently and efficiently with the expanded method of subtraction or understand mathematically what is happening should be introduced to the compact method including 4 digit numbers.

864 - 621 = 243

864

- 621

243

No adjustment

432 - 217 = 215

2

4 ~~3~~ 12

- 217

243

One adjustment: tens to ones

Next steps: Extend to two adjustments, e.g. 'tens to ones' and 'hundreds to tens'.

6 0 2 - 3 4 7 = 2 5 5

5 9
~~6~~ ~~0~~ 12
- 3 4 7
2 5 5

Dealing with zeros: OPTION 1

6 0 2 - 3 4 7 = 2 5 5

5 9 9 (+3)
- 3 4 7
2 5 2 (+3) **= 2 5 5**

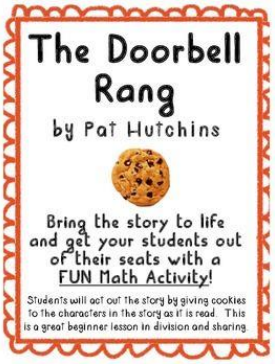
Dealing with zeros: OPTION 2

Next steps: Extend to numbers with **any number of digits up 1,000,000 (Y5) and 10,000,000 (Y6)**. When appropriate, use the compact method to subtract **two or more decimal fractions with up to 4 digits** and **either** 1, 2 or 3 decimal places including money and/or measures.

Mental Calculations: Division

Stage 1: FIRST STEPS IN DIVISION

FS



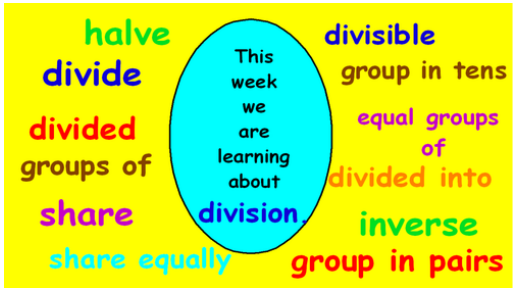
- Children **solve practical problems involving division in a real or role play context**. They make decisions about what to do, explain their thinking orally and, where appropriate, may begin to record the solution in their own way.

E.g. respond in practical situations to questions such as:
Sharing:
Can we share out these cakes fairly? How will we do it?
Can you share out this set of 15 stickers fairly between our group? Are there any left?
Grouping:
We have 12 smiley faces. How many groups of 4 can we make?
How many pairs of socks can I make using this set of 10?

Y1

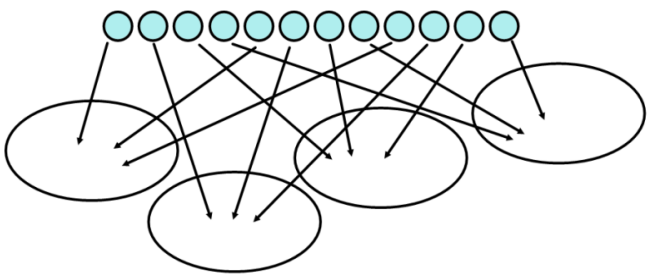
Stage 2: DEVELOPING UNDERSTANDING

- **Begins to use the related vocabulary associated with division** (shared equally between, in groups of, divided by, repeated subtraction etc).

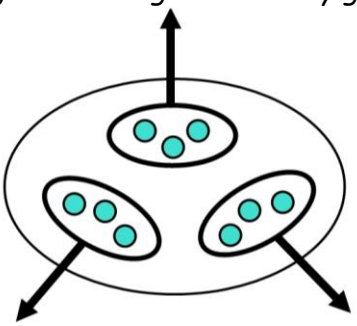


- **Understands the difference between the sharing method and the grouping method** of division and begins to recognise images associated with these methods.

E.g. 12 ÷ 4 = 3 by sharing objects equally:



E.g. calculating 9 ÷ 3 = 3 by grouping/repeated subtraction of objects:



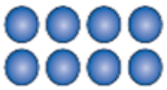
E.g. calculating 20 ÷ 5 = 4 grouping on fingers:

How many 5s in 20? = 20 ÷ 5: count in 5s up to 20 on fingers.

E.g. calculating 12 ÷ 3 = 4 by grouping on a number line

- Link to arrays that were practised to understand multiplication. Investigate the link between division and multiplication.

8 ÷ 2 = 4 1 2

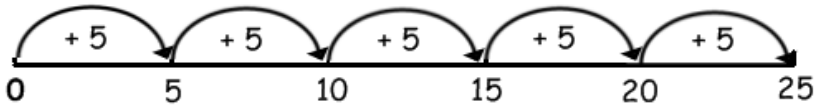


- Begins to describe and **record division number sentences**.
- Is beginning to **interpret problem solving situations as division calculations** and explain reasoning.

Stage 3: USING RECALL OF MULTIPLICATION FACTS AND PARTITIONING METHODS

- Once children have developed a **recall of multiplication facts**, steps in mental division can be recorded by jumping on an empty number line forwards or backwards.

E.g. 25 ÷ 5 = 5 using an empty number line:

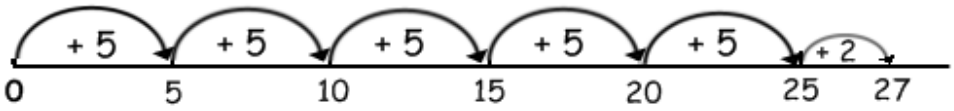


Next steps: Children can jump along the number line in other numbers relating to the

multiplication tables they have learnt. Children progress to examples that give rise to remainders. They will use recall of multiplication facts to enable them to jump more efficiently along the number line.

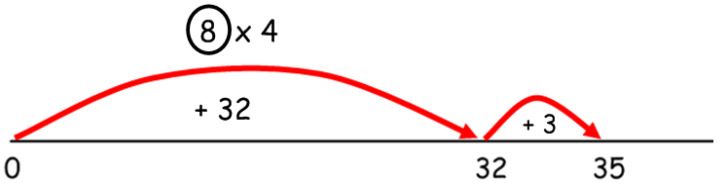
Y2 cont…

E.g. 27 ÷ 5 = 5 r 2 using an empty number line:

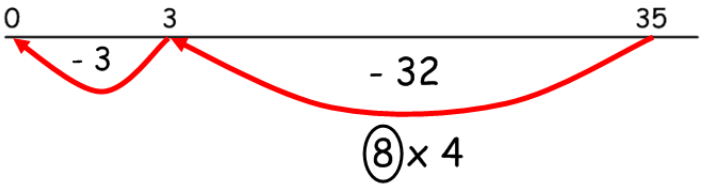


Y3

E.g. 35 ÷ 4 = 8 r 3 using an empty number line AND knowledge of multiplication facts:

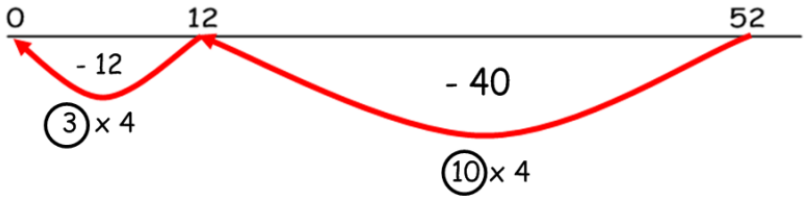


NB. This method can also be carried out using repeated subtraction.
E.g. 35 ÷ 4 = 8 r 3



<u>Next steps:</u> Children use knowledge of place value to help them when working with larger numbers.

- Children can use a **partitioning method** for numbers beyond x 12 of the divisor.
  E.g. 52 ÷ 4 = 13 using a blank number line:

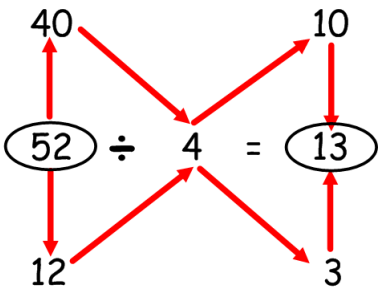


NB. A good understanding of this method will help children when they are introduced to the written method of chunking for division.

E.g. 52 ÷ 4 = 13 using informal jottings:

52 ÷ 4 = (40 + 12) ÷ 4
= (40 ÷ 4) + (12 ÷ 4)
= 10 + 3 = 13

or



Next steps: Children can subtract multiples of 10 from the starting number to make the calculation more efficient. Children progress to examples that give rise to remainders.

Children from Year Two onwards also need to understand and practise calculations families:

8 x 3 = 24

3 x 8 = 24

24 ÷ 3 = 8

24 ÷ 8 = 3

Written Calculations: Division

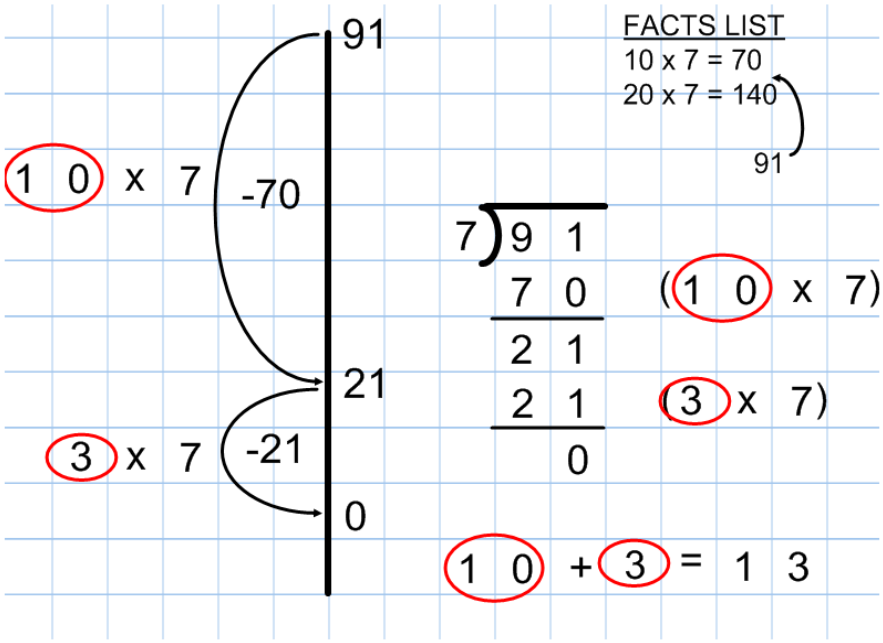
STAGE 4: INFORMAL EXPANDED METHOD: Chunking + Facts List

91 ÷ 7 = 13
Dividend ÷ divisor = quotient

Y3

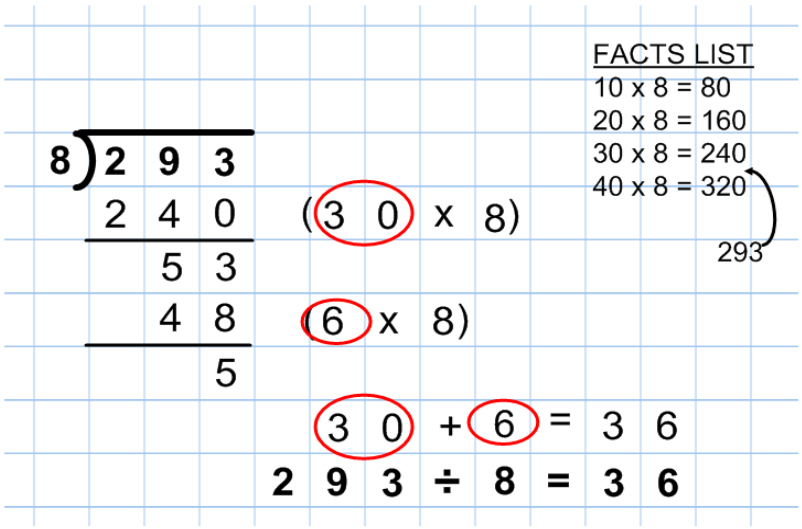
Method: Children use a **facts list** to 'trap the dividend' and help prevent them from creating extremely lengthy chunking calculations.

NB: It may be helpful to introduce this written method alongside a vertical number line as this builds on children's previous learning.



Short division:
TO ÷ O

Y4

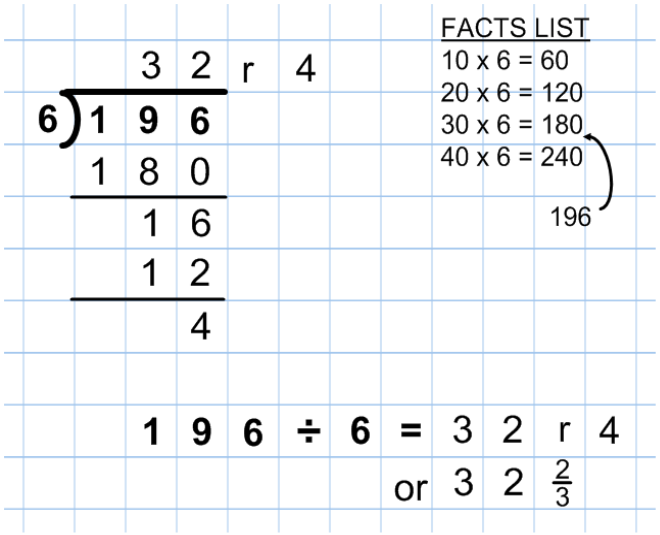


Short division:
HTO ÷ O

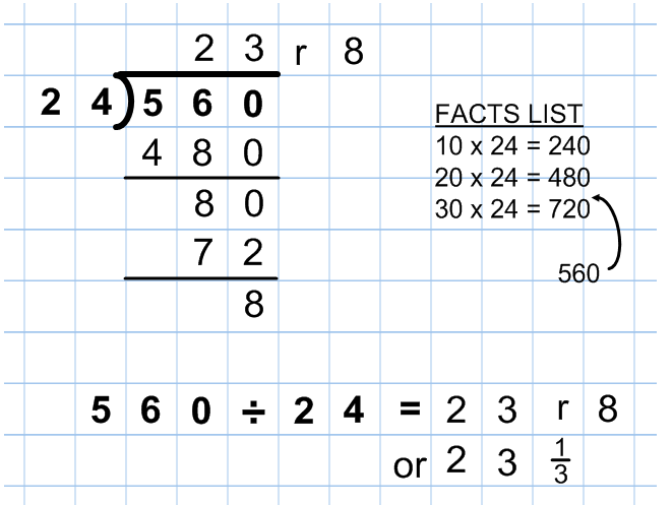
Progressively, generally have no remainders in Year 3, reminders as a whole number in Year 4 and remainders as a fraction or decimal where appropriate in Year 6. Although, if your children have a good understanding, they can be modelled the next stage.

STAGE 5: STANDARD METHOD

Y4/5



Short division:
HTO ÷ O and ThHTO ÷ O

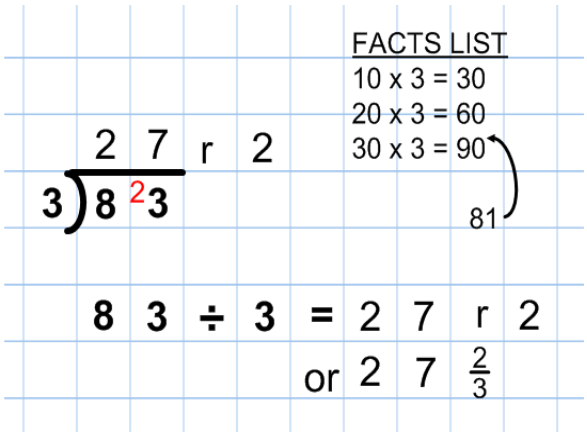


Long division:
HTO ÷ TO

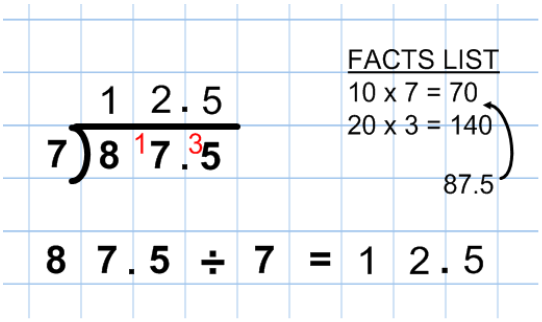
Y6

STAGE 6: COMPACT METHOD

Y4



Short division:
TO ÷ O

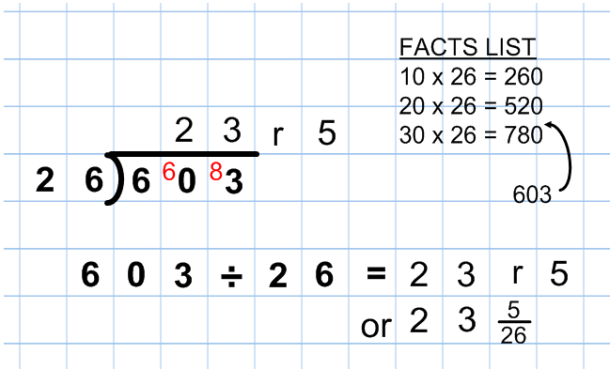


Short division:
TO.t ÷ O

Y5

Next steps: To extend to **TO.th ÷ O**

Y6



Long division:
HTO ÷ TO and
ThHTO ÷ TO

Next steps: To extend to larger integers.

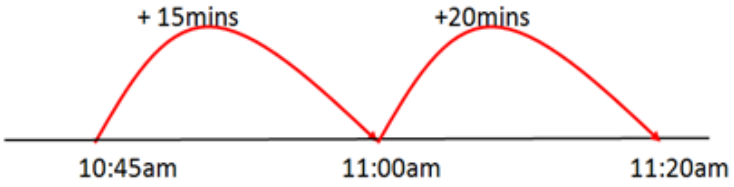
NOTE on using mental methods for problem solving

- Until children are confident when using a **written method** for addition, and have developed an understanding of decimals, they should be encouraged to add two or more digit sums of money or measures using the number line.

  E.g. £3.25 + 78p or 3.25m + 78cm

- Encourage all children to use the number line when solving problems involving time.

  E.g. Clare left her home at 10.45am and arrived at the shops 35 minutes later. What time did she arrive?



FINAL NOTE ON MENTAL METHODS: It is important that children's mental methods of calculation, including using the empty number line with increasingly large numbers, are practised and secured, **alongside** their learning and development towards a compact written method for each operation.

FINAL NOTE ON CALCULATIONS

By the end of Year 6, children will understand and use a **range of mental calculation methods**, and understand and use **one reliable written method for each of the 4 mathematical operations.**

- Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.
- Selection of calculation method will depend on the numbers involved.
- **Children should not be moved on to the next stage if they are not ready or they are not confident**.
- Children should be **encouraged to approximate their answers before calculating**.
- Children should be **encouraged to check their answers after calculation using an appropriate strategy**.

Policy updated: November 2015